7.1 The Playoffs
Graphing Inequalities

LEARNING GOALS

In this lesson, you will:

- Write an inequality in two variables.
- Graph an inequality in two variables.
- Determine which type of line on a graph represents a given inequality.
- Interpret the solutions of inequalities mathematically and contextually.

KEY TERM

- half-plane

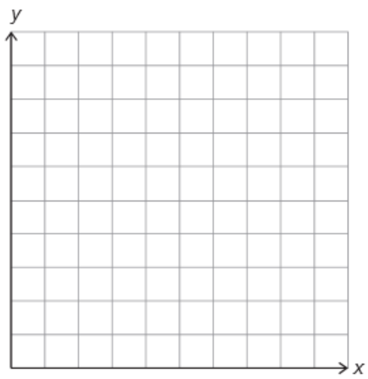
Basketball has come a long way to become the fast, action-filled sport it is today. In December 1891, a physical education teacher in Massachusetts was trying to keep his gym class active during the long winter. He wrote down some basic rules, nailed a peach basket to the wall, and basketball was born. At first, basketball was played with an association football, which is similar to a soccer ball. Because association footballs do not bounce very well, dribbling was not part of the game until the 1950s when the basketball we use today was introduced.

Originally, the scoring was pretty simple. If a player got the ball into the basket, their team got one point. The team with the most points at the end of the game won. The first official game was won with a score of 1–0 played on a court that was just half the size of the court basketball is played on today.

Why do you think changes were made to the way the game is played? Do you think basketball would be as popular if changes to the rules and regulations had not been made? Do you think any more changes would ever be made to this or any other sport? Why or why not?

PROBLEM 1 Crankin' It Up For the Playoffs!

1. Coach Purvis is analyzing the scoring patterns of a few players on his basketball team. Bena has been averaging 20 points per game from scoring on two-point and three-point shots.
   a. If she scores 6 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?
   b. If she scores 7 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?
   c. If she scores 7 two-point shots and 4 three-point shots, will Bena meet her points-per-game average?
2. Write an equation to represent the number of two-point shots and the number of three-point shots that total 20 points.
3. Graph the equation you wrote in Question 2 on the coordinate plane shown.



4. Coach Purvis believes that Danvers High School can win the district playoffs if Bena scores at least 20 points-per-game.

a. How can you rewrite the equation you wrote in Question 2 to represent that Bena must score at least 20 points-per-game?

b. Write an inequality in two variables that represents this problem situation.

Recall that an inequality is a statement formed by placing an inequality symbol ($<$, $\leq$, $>$, $\geq$) between two expressions. Recall that the forms of linear inequalities in two variables are:

$$ax + by < c$$
$$ax + by \leq c$$
$$ax + by > c$$
$$ax + by \geq c$$

5. Complete the table of values.

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total Points Scored
4	1	
6	1	
7	1	
8	2	
6	4	
9	5	

6. Use the data given in the table to plot the ordered pairs on the graph in Question 3. If the number of total points scored does not exceed Bena's points-per-game average, use an "x" to plot the point. If the number of total points scored meets or exceeds Bena's points-per-game average, use a dot to plot the point.

7. What do you notice about your graph?

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8. What can you interpret about the solutions of the inequality from the graph?

9. Choose an ordered pair (different from the ordered pairs in the table you completed) located above the graph and an ordered pair that is located below the graph. Does your interpretation of the situation seem correct? Explain your reasoning.

10. Shade the side of the graph that contains the combinations of shots that are greater than or equal to Bena's points-per-game average.

11. How do the solutions of the linear equation $2x + 3y = 20$ differ from the solutions of the linear inequality $2x + 3y \geq 20$?

12. Does the ordered pair (6.5, 5.5) make sense as a solution in the context of this problem situation? Explain why or why not?

PROBLEM 2 Line or Dash? Above or Below?

The graph of a linear inequality is a **half-plane**, or half of a coordinate plane. A line, determined by the inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions. These solutions are represented by shading the appropriate half-plane. If the inequality symbol is $\leq$ or $\geq$, the graph is represented by a solid line because the line is part of the solution set. If the symbol is $<$ or $>$, the graph does not include the line and is therefore represented by a dashed line.

1. Determine whether the graph of each inequality would be represented with a solid line or a dashed line on the coordinate plane.

   a. $y > 9 - x$

   b. $4x - 5y \geq 37$

   c. $x + \frac{2}{3}y \leq 6$

   d. $x + y < 4$

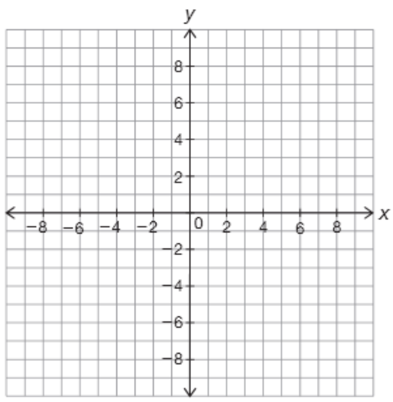
   e. $7x - y > 12$

   f. $y \leq 0.65x + 33$

2. Consider the linear inequality $y > 4x - 6$. The line that divides the plane is determined by the equation $y = 4x - 6$.

   a. Should the line representing this graph be a solid line or a dashed line? Explain your reasoning.

   b. Graph the inequality on the coordinate plane shown.



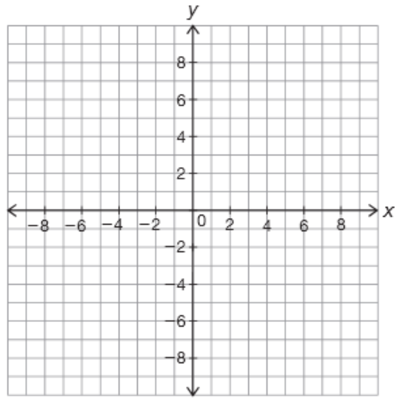
After you graph the inequality with either a solid or a dashed line, you need to decide which half-plane to shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains all the solutions and should be shaded. If (0, 0) is *not* a solution, then the half-plane that does not contain (0, 0) contains all the solutions and should be shaded.

   c. Is (0, 0) a solution? Explain your reasoning.

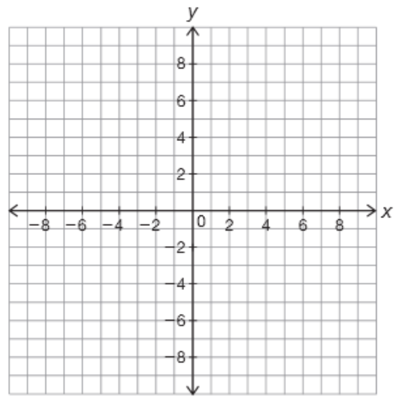
   d. Shade the correct half-plane on the coordinate plane.

3. Graph each linear inequality. Then shade the half-plane that contains the solutions.

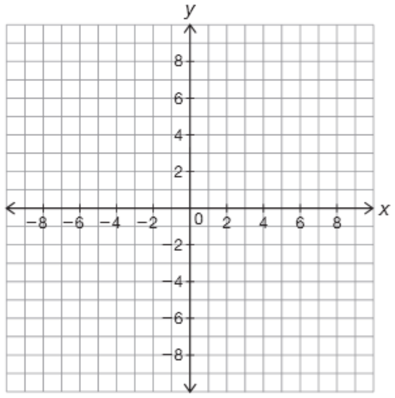
a. $y > x + 3$



b. $y \leq -\frac{1}{3}x + 4$



c. $2x - y < 4$



PROBLEM 3 I Just Can't Decide!

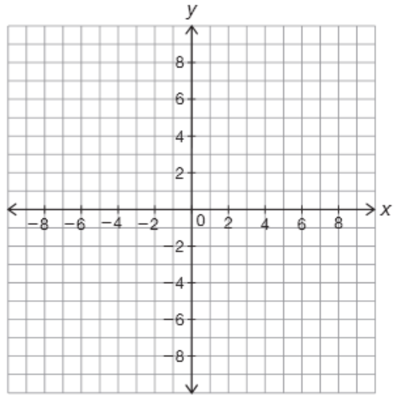
Your cousin's graduation party is a video game party at the arcade! Each person at the party receives a card with 50 points on it to play the games in the arcade. One of your favorite driving games uses 12 points per game. You also like a basketball game that uses 8 points per game. You want to determine how many times you can play each of these games without exceeding the 50 points on the game card.

1. Write an inequality to represent the problem situation. Define your variables.

2. Complete the table that represents different numbers of times you play the driving game and the basketball game and the total numbers of points used.

Number of Driving Games Played	Number of Basketball Games Played	Total Number of Points Used
0	5	
1	3	
2	3	
2	4	
3	2	
3	3	
4	0	
4	1	

3. Graph the inequality you wrote in Question 1 on the coordinate plane shown. Then use the data in the table to plot the ordered pairs.



4. Is the ordered pair (−1, 8) a solution of the inequality you wrote for this problem situation? Why or why not.

5. Is the ordered pair (7, −3) a solution of the inequality you wrote for this problem situation? Why or why not.

6. What can you interpret about the solution set from the graph of this problem situation? Explain your reasoning.

Be prepared to share your solutions and methods.